FRAGILITY ASSESSMENT THEORY AND TEST PROCEDURE

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In order to adequately and economically package goods for shipment, it is necessary to know both the input environment and the characteristics of the product. The package, then, must account for the difference between the environmental stresses which occur and the product's ability to withstand abuse. If either of these areas is overlooked, excessive damage in shipment or excessive packaging costs can result.

This report will examine techniques which can be used to accurately assess product fragility from the standpoint of shock and vibration. Although other environmental parameters such as temperature, humidity, etc. must sometimes be considered, it is generally agreed that most shipping damage is caused by shock and/or vibration.

I. Shock Fragility Assessment

The severest shocks likely to be encountered during shipment result from dropping of the package onto a floor, dock, or platform.

Consider first the process by which the effect of the abrupt deceleration of the outer package at the termination of a drop is communicated to the packaged item. The nearly instantaneous velocity change which takes place at the outer surface of the package upon striking the floor is accompanied by local accelerations of many thousands of g's. The compliance of the outer package, the cushioning material, and the inner package (if any) transforms the pulse delivered to the packaged item so that the maximum acceleration is greatly reduced and the time required to attain this maximum is many times as long. The situation is represented qualitatively in Figure 1. The maximum cushion deformation is assumed to occur at B. The corresponding ordinate BM is generally close to the maximum for the packaged item. The shaded areas under the two curves must be substantially equal, since each of these areas corresponds to the striking velocity* Because the cushioning material exhibits some elastic recovery, upward acceleration of the packaged item continues until point C is reached.

Some additional oscillation of the packaged item will generally occur, but the accompanying accelerations are quite small compared with the first maximum. A useful simplification for analysis and testing results from assuming that the damaging effects result solely from that portion of the curve between A and C. Thus the input motion is simplified as a single acceleration pulse. The shaded area, A to B, is equal to the striking velocity. The unshaded area, B to C, is equal to the rebound velocity. The ratio of rebound velocity to striking velocity is called the coefficient of restitution e. Energy considerations establish that e must lie between 0 (fully plastic impact, no rebound) and 1 (fully elastic impact).

*There is an area difference, usually negligible, because the packaged item continues to accelerate downward until the cushion force exceeds the item weight.

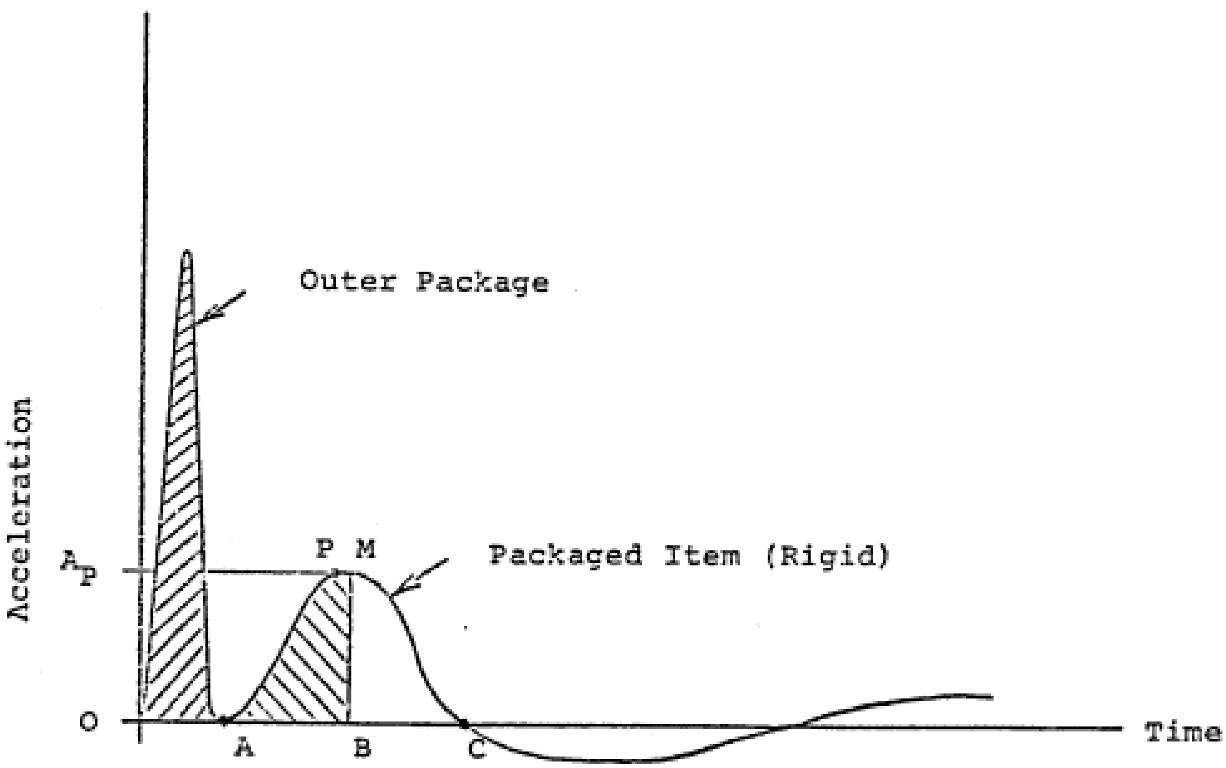


Figure 1. <u>Drop Test Accelerations</u>

Usual practice in package design assumes that the maximum acceleration A_p alone measures the severity of shock. If this is less than the rated fragility of the item, a margin of safety against damage is assumed to exist. This method neglects the influence of item flexibility upon the damaging effects of the shock. Because any real item has distributed mass and flexibility, it will undergo elastic (and possibly inelastic) deformations during a shock. Correspondingly, maximum accelerations will not be the same throughout the packaged item.

It is readily apparent that a fully rational analysis of the response motion of an actual packaged item is not practical. There is available, however, a simplified mathematical model which affords a substantial improvement over the rigid model without introducing unmanageable complexity. .This model is the basis for the widely-used shock spectrum. Briefly, the model for the packaged item consists of a rigid mass m_1 to which a second mass m_2 is attached by a spring of stiffness K (see figure 2). The rigid mass m_1 is assumed to represent the bulk of the item. The small mass m_2 and spring K represent a critical component and its stiffness (flexibility). The fragility of this model is characterized by the maximum allowable acceleration of the critical component (acceleration of m_2). Because m_2 is much smaller than m_1, its effect on the motion of m_1 may be neglected. Accordingly, the system is analyzed by assuming that m_1 undergoes a specified acceleration vs. time history and the resulting maximum acceleration of m_2 is determined. Results of the foregoing analysis are usually presented as a shock spectrum. For a pulse of specified shape (e.g. a half-cycle sine wave) the ratio of the peak acceleration of m_2 to the maximum acceleration of the input pulse is plotted versus the product of pulse duration by the natural frequency of the critical component. Such a curve is shown in Figure 3. Symbols used are defined as follows:

A_p = peak acceleration of input pulse

A_C = peak acceleration of critical component

τ_e = effective pulse duration = $\dfrac{\text{velocity change}}{A_p}$

f_c = natural frequency of critical component

$$= \frac{1}{2\pi} \sqrt{\frac{K}{m_2}}$$

Examination of Figure 3 discloses that, for a given shock (specified τ_e and A_p), the peak acceleration of the critical component depends strongly upon component frequency. In particular, if $f_c \tau_e < \frac{1}{6}$ the component peak acceleration depends only on the velocity change $V = A_p \tau_e$ of the pulse. Such a low frequency component is, in a sense, its own shock isolator and it benefits little, if at all, from the cushioning. On the other hand, for components with $f_c \tau_e > \frac{1}{6}$, the component peak acceleration exceeds that of the pulse. The ratio may vary from 1 to about 1.8, depending on component frequency.

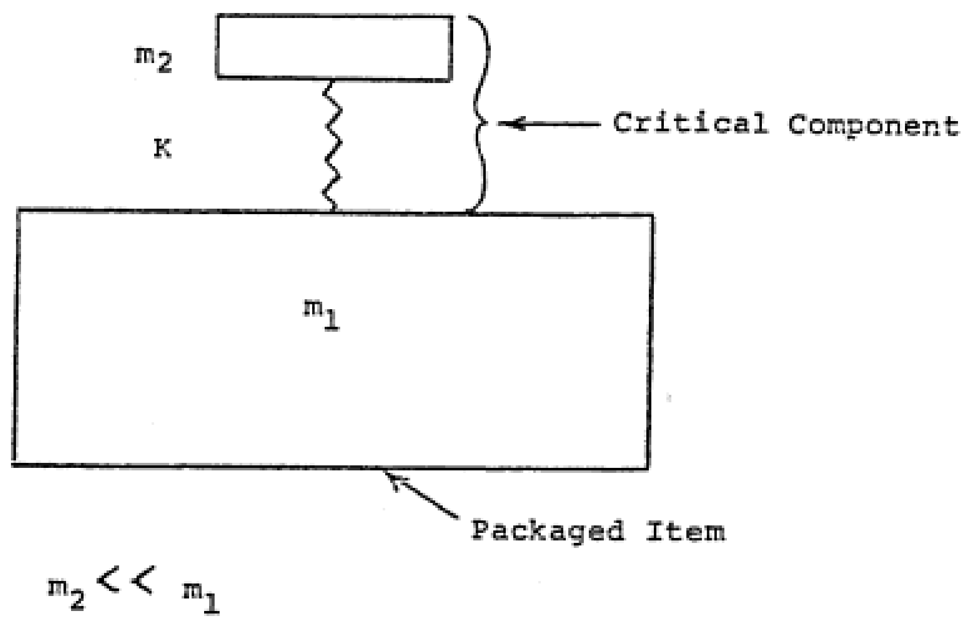


Figure 2. Mathematical Model for Packaged Item

To make rational use of the shock spectrum the designer needs much information not usually available. For the equipment he needs to know:

A_{cs} = maximum safe peak acceleration of critical component;

f_c = natural frequency of critical component.

It should be observed that there may be many fragile components, each of which might be critical for a particular pulse. To verify the adequacy of a particular design the following are needed:

A_p = peak acceleration transmitted to item;

τ_e = effective duration of acceleration pulse.

If the acceleration pulse were always a half-sine, the above information would suffice. Unfortunately, this is not the case and the shock spectrum is sensitive to pulse shape. In Figure 4 the shock spectrum curves for four different acceleration pulses are shown. Examination discloses that the maximum component acceleration may range from about 0.9 A_p to 2.0 A_p for shocks of different shapes, even though τ_e and A_p and fixed.

A realistic appraisal of the data requirements makes evident the impracticality of using a fully rational version of shock spectrum analysis for routine package design. A simpler, and somewhat less accurate, procedure is needed. Specifically, it is proposed that fragility tests be based on shock spectra, but that cushion selection may continue to be based on drop height, static stress, and peak acceleration. Details are given in the following paragraphs.

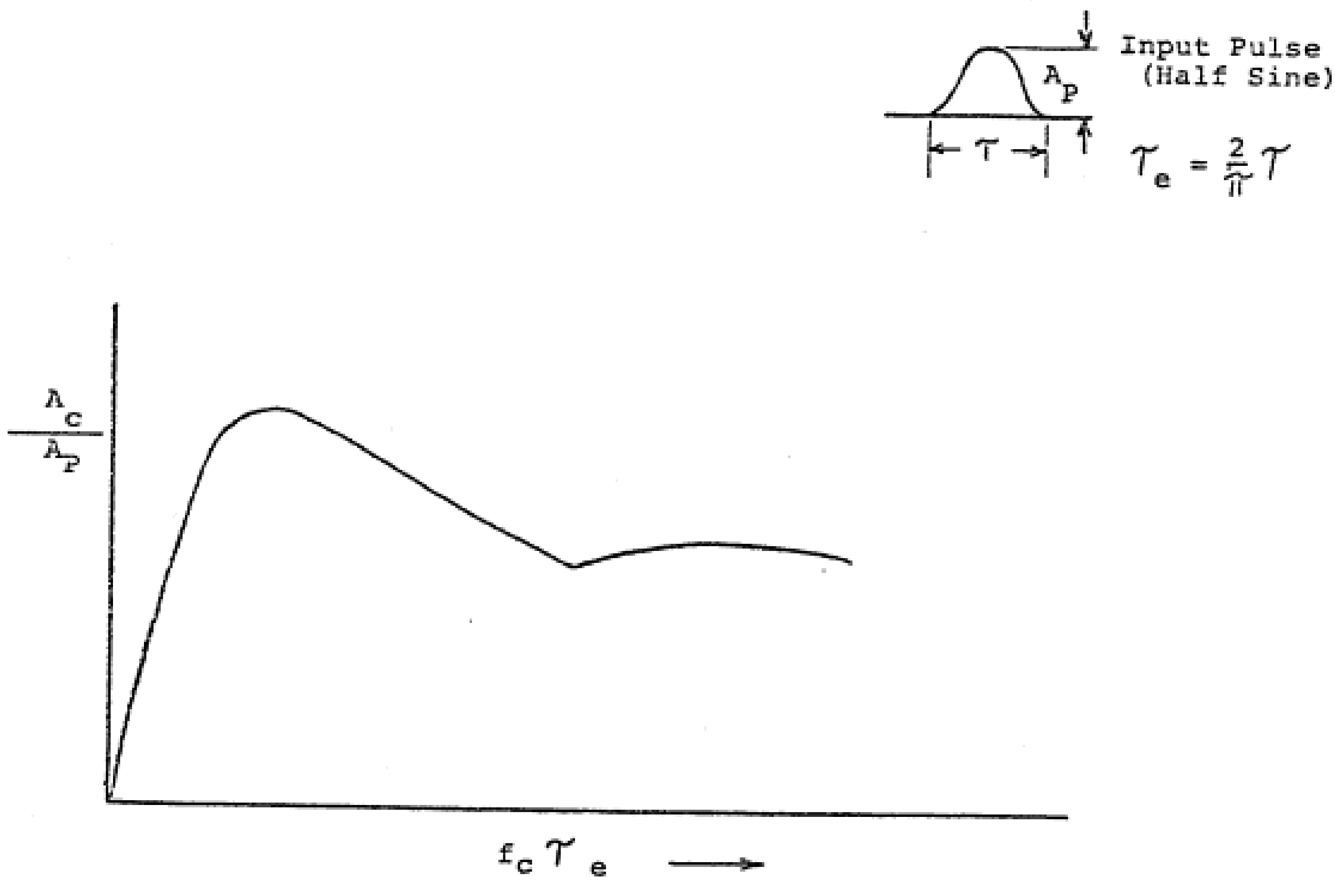


Figure 3. Shock Spectrum, Half Sine Pulse

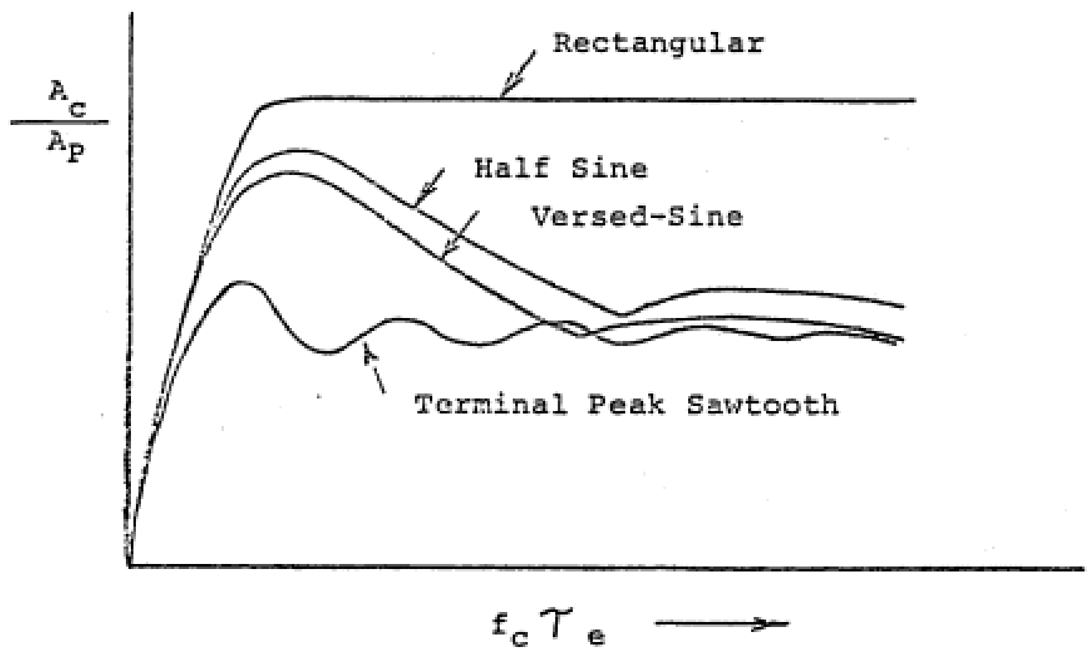


Figure 4. <u>Shock Spectra, Various Pulses</u>

Comparison of the shock spectrum curves for various pulse shapes (Figure 4) reveals that the rectangular pulse curve provides an upper bound. Thus, if an item is subjected to a shock pulse of given peak acceleration A_p and effective duration τ_e the peak acceleration of any component will not exceed that which would result from a rectangular pulse having the same A_p and τ_e. Accordingly, it is proposed that item fragility tests be performed using rectangular pulses. The only complication in this procedure is that the effective duration τ_e to be used is not initially known.

In order to determine an appropriate pulse duration for tests it is pertinent to recall that:

$$V = A_p \tau_e \qquad (1)$$

where V is the velocity change (pulse area). Thus $\tau_e = V/A_p$ and it is uniquely determined by these two parameters. The velocity change in a drop test may be expressed as:

$$V = (1 + e) \sqrt{2gh} \quad (2)$$

where e is the coefficient of restitution, g is the acceleration of gravity, and h is the drop height. Now, as has been observed earlier, e must be between 0 and 1. Accordingly, the V for a given drop height is known within the limits of uncertainty on e and:

$$\sqrt{2gh} \leq V \leq 2 \sqrt{2gh} \quad (2)$$

Ignoring this uncertainty in V for an actual drop, a test procedure may be used based upon controlled values of V and A_p.

For a chosen V, drops are made for successively increasing peak pulse accelerations A_p. The item is inspected for damage following each drop. The maximum peak pulse acceleration A_{cs} that can be sustained without damage is taken to be the fragility measure of the item <u>at that velocity change V</u>.

Customary practice ignores the indicated dependence on V. It is instructive to explore the nature of this dependence.

Consider the shock spectrum for a rectangular pulse shown in Figure 5.

It is easy to show that the segments of OA, AB and BC are represented with adequate accuracy as follows:

$$\text{OA:} \quad \frac{A_c}{A_p} = 2\pi f_c \tau_e, \quad 0 \leq f_c \tau_e < \frac{1}{6}\ ; \tag{3}$$

$$\text{AB:} \quad \frac{A_c}{A_p} = 2 \sin(\pi f_c \tau_e), \quad \frac{1}{6} \leq f_c \tau_e \leq \frac{1}{2}\ ; \tag{4}$$

$$\text{BC:} \quad \frac{A_c}{A_p} = 2, \quad \frac{1}{2} \leq f_c \tau_e. \tag{5}$$

Assume now that the critical component has a specified natural frequency f_c and that it may safely sustain a peak acceleration A_{cs} without damage. Substituting A_{cs} for A_c in Equation 3, replacing $A_p \tau_e$ by V (Equation 1) and solving for V gives:

$$V_\ell = \frac{1}{2\pi} \quad \frac{A_{cs}}{f_c}\ , \tag{6}$$

where a subscript ℓ has been appended to V. It is easy to show that a pulse for which $V \leq V_\ell$ will result in a peak component acceleration $A_c \leq A_{cs}$. It is evident from inspection of Equation 5 that any shock for which $A_p \leq A_{pr}$ will likewise give $A_c \leq A_{cs}$, where:

$$A_{pr} = \frac{1}{2} A_{cs} \tag{7}$$

Note also that:

Point A: $V = V_\ell$, $A_p = A_{cs}$; (8)

Point B: $V = \frac{\pi}{2} V_\ell$, $A_p = \frac{1}{2} A_{cs}$ (9)

Finally, substitution in Equation 4 gives the relation:

$$\frac{A_{cs}}{A_p} = 2 \sin \frac{\pi f_c V}{A_p}, \tag{10}$$

which defines the relation between A_p and V in the segment between A and B.

Using the results of Equations 6, 7, 8, 9, 10, Figure 6 is plotted.

In Figure 6, the curve O A B C is the image in the V, A_p plane of the shock spectrum curve with $A_c = A_{cs}$ and f_c = constant. (Point O lies at $V = V_\ell$, $A_p = \infty$ in the V, A_p plane.) For shocks on the curve or in the unshaded region to the left and below, $A_c \leq A_{cs}$ and no component damage will result. Conversely, shocks corresponding to points in the shaded region give $A_c > A_{cs}$ and damage will result.

The practical importance of the vertical portion of the damage boundary depends upon the drop height against which protect is to be afforded. In some cases, the item may have a sufficiently

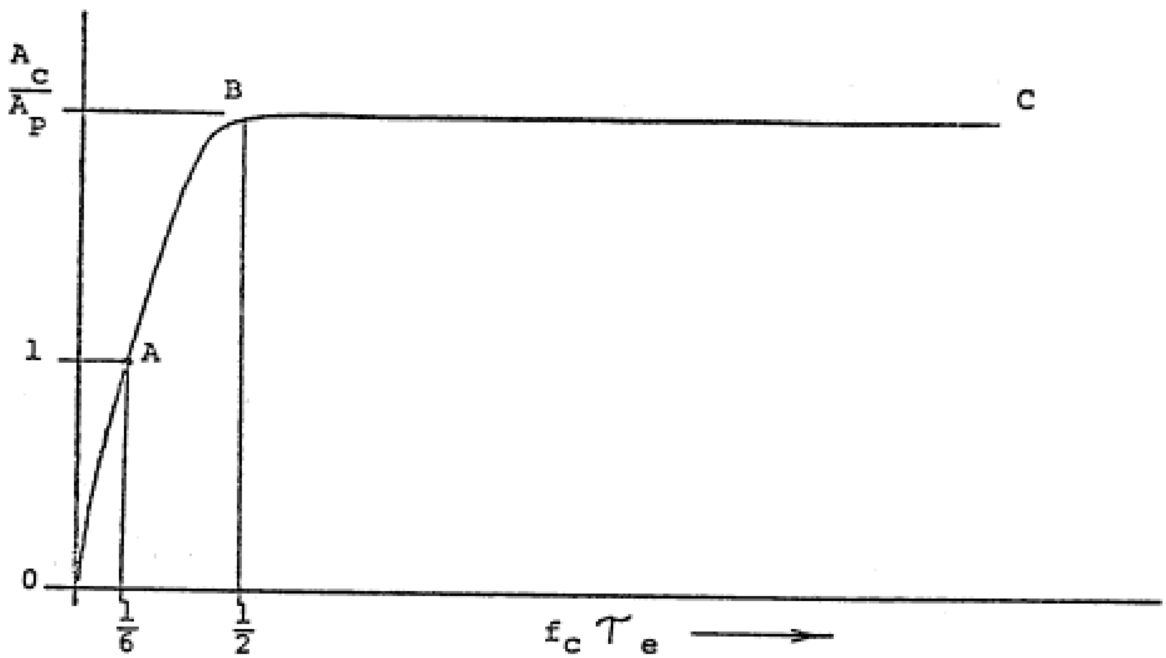


Figure 5: Shock Spectrum, Rectangular Pulse

high V_ℓ so that no cushioning is needed to prevent damage during drops from the design height.

To establish the complete damage boundary by test requires a shock test machine which is equipped with suitable shock programmers to control the acceleration-vs.-time pulses.

As already described, tests at constant V and progressively increasing A_p will establish the ordinate A_{p_r} for the right hand portion (bottom line) of the damage boundary. A second sequency of tests with successively increasing V will establish V_ℓ , the velocity change defining the left-hand portion of the damage boundary. It has been shown that the transition curve is tangent to the vertical line $V_\ell = V$ at $A_p = 2A_{p_r}$, and is tangent to the horizontal line $A_p = A_{p_r}$ at $V = 1.57\ V_\ell$. The transition curve may be plotted from calculated points or a square corner may be substituted.

It should be noted that only a limited range of velocity changes V is of practical interest. The lower limit is found from Equation 2 using the minimum H of interest with e = 0. The upper limit is given by the maximum H and e = 1. Clearly there is no reason to conduct fragility tests outside this range of velocity change V.

II. Vibration Fragility Assessment

While the shock environment may be neatly characterized in terms of drop height, the vibration environment is unquestionably random in nature. This implies that accurate environmental simulation requires random vibration testing-- and this is indeed true. But the purpose of fragility assessment is not necessarily to reproduce the environment, but to analyze and reproduce its damage-causing properties. The approach, then, is to investigate the most probable failure modes of the product and their relationship to the environment.

It is generally agreed that the transportation vibration environment has no significant high acceleration content which would directly cause damage due to non-resonant inertial loading. Damage is most likely to occur when some element or component of the product has a natural frequency which is excited by the environment. If this "tuned" excitation is of sufficient duration, component accelerations and displacements can be amplified to the failure level.

The response of a product or component to input vibration may be represented by a curve having a shape generally like that of Figure 7 (this is a familiar "transmissibility curve" for a single-degree of-freedom system).

A_R = Response acceleration of the product or component

A_I = Input acceleration

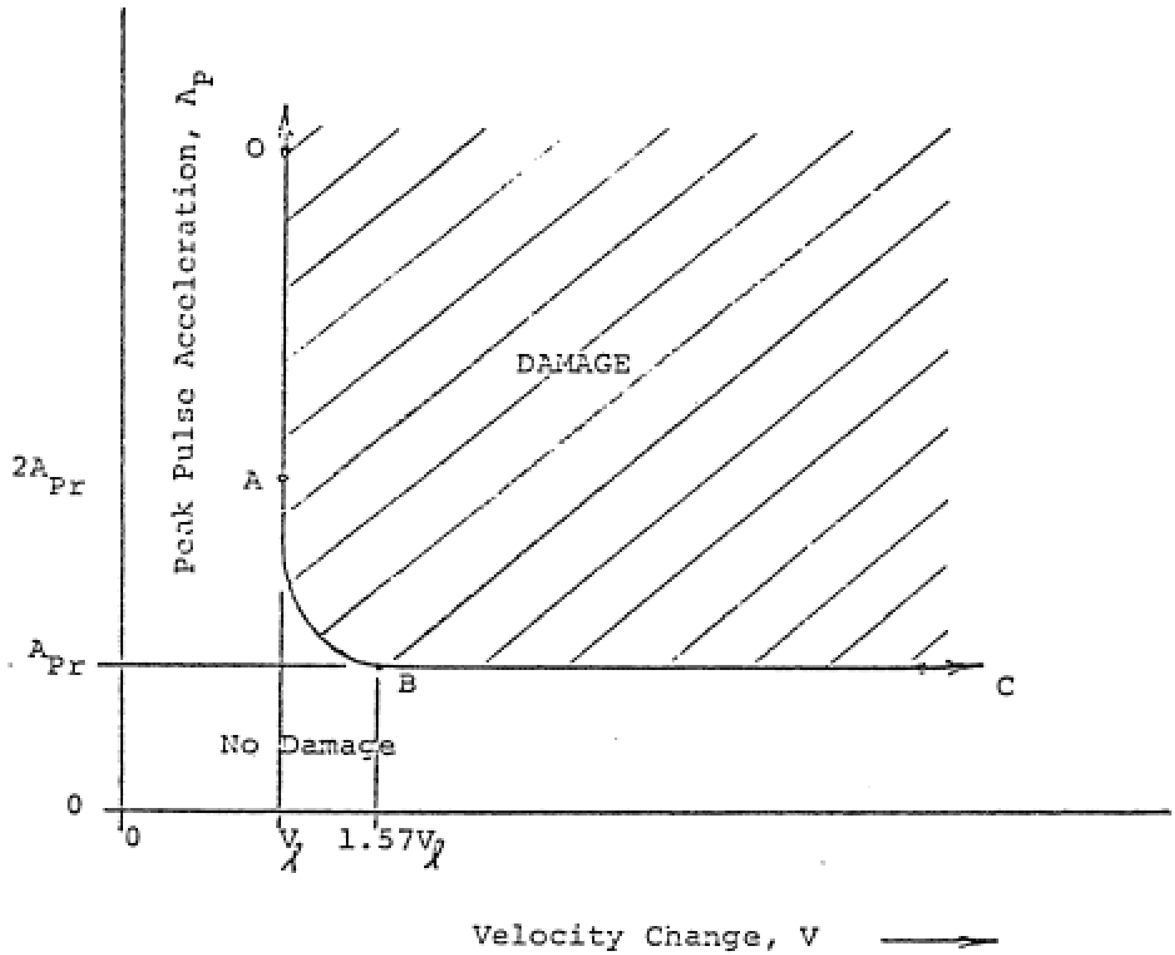
f = Input frequency

f_R = Resonant frequency of the product or component

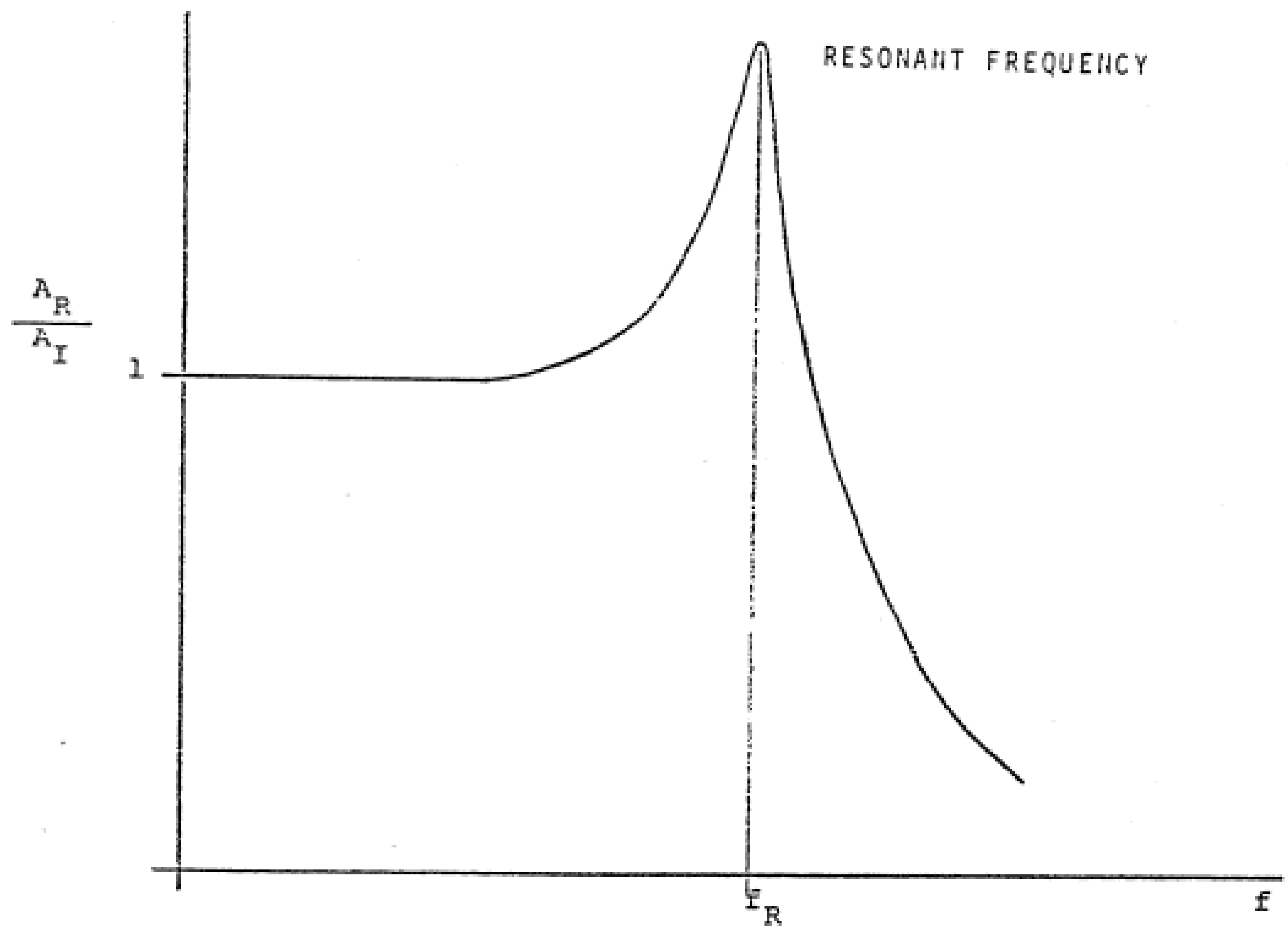
It can be seen that for very small values of $\frac{f}{f_r}$,

$A_R/A_I = 1$. For very large values of f/f_r, $\frac{AR}{A_I} = 1$. But at or near $f = f_r$, $A_R/A_I \gg 1$. The exact magnitude of the peak amplification depends upon damping in the system, but values or 5 or 10 are not uncommon. This is the frequency region where, for that particular product or component, damage is most likely to occur. Vibration fragility assessment consists of identifying these critical components and frequencies.

The tests require a vibrator which is capable of producing single-axis sinusoidal motion over a frequency range sufficiently wide to encompass the frequencies which occur during transportation. The acceleration amplitude is not critical as long as it is great enough to excite identifiable resonances but not so great as to product damage in short-duration tests.



DAMAGE BOUNDARY



VIBRATION TRANSMISSIBILITY
CURVE